Supporting Information

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SI Text

Modeling Mode Competition and Lasing Thresholds in Chaotic Cavities

The ability to achieve highly multimode lasing at a moderate pump value depends on two factors: a relatively uniform $Q$ distribution and minimized mode competition. We used the steady-state ab initio laser theory (SALT) to understand the role of both of these factors in D-shaped cavities and optimize the geometry to enable highly multimode lasing. SALT first calculates the noninteracting modal thresholds and then updates these thresholds in the presence of modal interactions as the pumping of the gain medium is increased. The noninteracting lasing thresholds incorporate both effects of the mode decay rate and the distance of its frequency from the center of the gain spectrum and can be calculated as

$$D_n = \eta_n \left( \frac{k_n - k_a + i \gamma_a}{\gamma_a} \right). \tag{S1}$$

where $D_n$ is the noninteracting threshold of the $n$th mode, $k_n$ is lasing mode frequency, $k_a$ is the center frequency of the gain spectrum, $\gamma_a$ is the width of the gain spectrum, and $\eta_n$ is the threshold constant flux (TCF) eigenvalue of the associated cavity resonance. The TCF eigenvalue can be calculated for a homogeneous dielectric cavity with uniform pumping, such as the ones studied here, as

$$\eta_n = \eta_c \left( \frac{(k_n - i \gamma_n/2)^2}{k_n^2} - 1 \right). \tag{S2}$$

where $\eta_c$ is the passive cavity dielectric constant and $\gamma_n$ is the intensity decay rate of the $n$th mode, which can be calculated from the $Q$ factor,

$$Q_n = \frac{k_n}{\gamma_n}. \tag{S3}$$

Qualitatively speaking, for the high-$Q$ cavities studied here, $\eta_n$ contains information about the decay rate of the $n$th lasing mode, which must be compensated for to achieve lasing, whereas the term in the parentheses in Eq. S1 adjusts this value due to the distance between the lasing mode frequency and the center of the gain spectrum. Although the TCF eigenvalue and the distance from the center of the gain spectrum are both complex quantities, the noninteracting threshold is reached when the imaginary components of these factors cancel exactly, allowing for a positive, real threshold, $D_n$. Above threshold, the lasing modes satisfy a set of coupled, nonlinear, differential equations. SALT facilitates the solution of these equations by assuming that the lasing modes are operating in the continuous wave regime.

Fig. S1A shows the $Q$ distribution for each of the cavity modes, clearly showing that the D-shaped cavities exhibit a far more uniform $Q$ distribution than the circular microcavity. In Fig. S1B, we show the lasing thresholds for each cavity type without normalization. Note that Fig. 1E in the main text is a normalized version of these same data. As expected, the circular microcavity shows the lowest lasing threshold, because it supports the highest $Q$ mode. However, in the circular cavity the threshold increases dramatically for higher-order modes and the pump value required to turn on all of the first 10 modes is actually higher in the circular cavity than in the D-shaped cavity with $r_0 = 0.5R$. The ability to achieve highly multimode lasing at a moderate pump value depends on two factors: a relatively uniform $Q$ distribution and minimized mode competition. We used the steady-state ab initio laser theory (SALT) to understand the role of both of these factors in D-shaped cavities and optimize the geometry to enable highly multimode lasing. SALT first calculates the noninteracting modal thresholds and then updates these thresholds in the presence of modal interactions as the pumping of the gain medium is increased. The noninteracting lasing thresholds incorporate both effects of the mode decay rate and the distance of its frequency from the center of the gain spectrum and can be calculated as

$$D_n = \eta_n \left( \frac{k_n - k_a + i \gamma_a}{\gamma_a} \right). \tag{S1}$$

where $D_n$ is the noninteracting threshold of the $n$th mode, $k_n$ is lasing mode frequency, $k_a$ is the center frequency of the gain spectrum, $\gamma_a$ is the width of the gain spectrum, and $\eta_n$ is the threshold constant flux (TCF) eigenvalue of the associated cavity resonance (1). The TCF eigenvalue can be calculated for a homogeneous dielectric cavity with uniform pumping, such as the ones studied here, as

$$\eta_n = \eta_c \left( \frac{(k_n - i \gamma_n/2)^2}{k_n^2} - 1 \right). \tag{S2}$$

where $\eta_c$ is the passive cavity dielectric constant and $\gamma_n$ is the intensity decay rate of the $n$th mode, which can be calculated from the $Q$ factor,

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Qualitatively speaking, for the high-$Q$ cavities studied here, $\eta_n$ contains information about the decay rate of the $n$th lasing mode, which must be compensated for to achieve lasing, whereas

Fig. S1. (A) Calculated quality factors for the 20 highest-Q modes in three D-shaped cavities and a circular cavity of the same radius. The cavity parameters are identical to those in Fig. 1. The D-shaped cavity with \( r_0 = 0.5R \) has the most uniform Q distribution. (B) Calculated lasing thresholds without mode competition (dashed lines) and with mode competition (solid lines) for three D-shaped cavities and a circular cavity. Although the circular cavity has the lowest threshold for the first few lasing modes, it has the highest thresholds for the 7–10th lasing modes due to uneven Q distribution and strong mode competition for gain. In contrast, the D-shaped cavities, and in particular the optimal one with \( r_0 = 0.5R \), exhibit much more uniform thresholds for the first 10 lasing modes. Thus various D cavities are distinguished primarily by the different strengths of mode competition in each.

Fig. S2. Characterization of four lasers with different cavity shapes fabricated on the same wafer. The emission spectra below and above lasing threshold are shown for each cavity along with a top-view microscope image of the cavity and an image of the speckle pattern formed at the end of a multimode fiber. The speckle contrast, \( C \), and the number of mutually incoherent lasing modes, \( M \), are listed for each laser. Strong mode competition in the Fabry–Perot cavity (A) and the circular microdisk (B) limit lasing to \( \sim 3 \) and \( \sim 15 \) modes, respectively. On the other hand, the chaotic cavity lasers support highly multimode lasing, with the 100-μm radius D-shaped cavity (C) supporting \( \sim 100 \) modes and the 500-μm radius D-shaped cavity (D) supporting \( \sim 1,000 \) modes.