Supplemental Document



# Distributed Brillouin fiber laser sensor: supplement

JOSEPH B. MURRAY,<sup>1,\*</sup> D ALEX CERJAN,<sup>2</sup> AND BRANDON REDDING<sup>1</sup>

<sup>1</sup>U.S. Naval Research Laboratory, 4555 Overlook Ave. SW, Washington 20375, USA <sup>2</sup>Center for Integrated Nanotechnologies, Sandia National Laboratories, Albuquerque, New Mexico 87123, USA

\*Corresponding author: joseph.murray@nrl.navy.mil

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# Automated mode amplitude balance

An automated routine was used to adjust the transmission through  $EOM_2$  in order to establish multimode lasing with comparable mode amplitudes. We first set the transmission for each mode to be equal. We then recorded a measurement containing 20 pulse trains and calculated the average amplitude of each lasing mode. Next, we identified any modes with amplitude 25% above or below the mean amplitude. We then made a small (1%) adjustment to the transmission for these modes (i.e. the transmission was increased for modes that were too weak and decreased for modes that were too strong). This routine proceeded iteratively until the amplitude of each lasing mode was within  $\pm 25\%$  of the mean.

## **Demodulation details**

I/Q demodulation was performed in software to obtain the frequency of the lasing modes. The nominal demodulation frequency for each mode was first estimated by finding the peak of the Fourier transform of the interference signal. We then used a Hann window to select the mode of interest from each pulse train and recovered the amplitude and phase of the lasing mode via standard I/Q demodulation using a finite impulse response low-pass filter. The low-pass filter was set to have a 3 dB point at the nominal demodulation frequency. The slope of the phase was then fit via linear regression using the 40 ns with the largest amplitude (corresponding to the pulse duration). Finally, the lasing frequency was estimated from the slope of the phase (plus the demodulation frequency), as discussed above, and changes in the lasing frequency were converted to strain in the fiber assuming a standard response of 50 kHz/ $\mu\epsilon$ .

# **Doppler shift compensation**

In this section, we describe a technique to compensate for the Doppler frequency shift created from dynamic strain [1,2]. To do this, we added a reference interferometer using a second laser, which was coupled in and out of the FUT using a WDM filter (see figure S2). Using this reference interferometer, we recorded the time-varying phase accumulated through the 400 m FUT,  $\phi_{ref}(t)$ . We then calculated the single-pass Doppler frequency shift experienced by light passing through the FUT:  $\Delta f_{ref}(t) = [d\phi_{ref}(t)/dt]/(2\pi)$ . To correct for the impact of this Doppler shift on the lasing modes, we also needed to measure the strength of the gain pulling effect on each mode. This was accomplished by wrapping a short section of the FUT on a reference PZT and introducing a calibration signal (the reference PZT did not overlap with any of the sensor positions). We then calculated the complex ratio,  $\gamma_n$ , between the frequency deviation observed for each lasing mode,  $\Delta v_n(F_{PZT,ref})$ , due to the reference PZT modulation, and the single-pass frequency deviation measured using the reference interferometer,  $\Delta v_{ref}(F_{PZT,ref})$ :

$$\gamma_n = \frac{\Delta v_n (F_{PZT,ref})}{\Delta v_{ref} (F_{PZT,ref})}$$
(S1)

where  $\Delta v_n$  is the Fourier transform of  $\Delta f_n$ , *n* enumerates the modes, and  $F_{PZT,ref}$  is the frequency of the calibration tone. This ratio provided a scale factor, which accounts for the increased sensitivity to strain-induced Doppler shifts of the lasing modes. This scale factor varied slightly from mode to mode but was ~3.5 in our experiments using 40 ns pump pulses. To perform the correction, we create the analytic signal using the Hilbert transform of the

reference and lasing mode frequency shifts (to allow for a phase difference between the two signals). The corrected signal is given by:

$$\Delta f_{n,corrected} = H^{-1} \left\{ H\{\Delta f_n\} - \gamma_{n,eff} H\{\Delta f_{ref}\} \right\}$$
(S2)

where H is the transform to the analytic signal and  $H^{-1}$  is the inverse transform. This approach added negligible noise to the measurements since the noise in the CW reference interferometer was much lower than the noise in the measured lasing modes. While this technique adds some experimental complexity, it shows that cross talk due to strain-induced Doppler shifts is a tractable problem and may be addressed more efficiently in a future work.



**Fig. S1.** The experimental setup shown in Fig. 2 was modified to include a reference interferometer probed by a separate laser. This interferometer was used to suppress cross talk due to strain induced Doppler shifts. The acousto-optic modulator (AOM) introduced a 55 MHz frequency shift enabling I/Q demodulation, while the WDM was used to reject the pump light. A reference PZT was also added after the FUT to introduce a calibration tone.

#### Gain pulling spread spectrum for increased bandwidth

One of the attractive features of Brillouin lasers is their ability to suppress the frequency noise of the pump laser, enabling narrow linewidth lasing [3]. However, in the Brillouin laser sensor presented here, this phenomenon can also reduce the sensor response to changes in the Brillouin resonance. To evaluate the impact of this effect on the Brillouin laser sensor, we considered a simple model where a lasing mode circulates with power spectrum,  $P_{lase}(\Delta f)$ , where  $\Delta f$  is the offset from the center of the Brillouin gain spectrum. During each round trip, this mode experiences loss,  $\alpha$ , and Brillouin gain,  $e^{G_{SBS}(\Delta f)}$ . Using a standard model for saturation, we can express the Brillouin gain as [4]:

$$G_{SBS}(\Delta f) = \frac{g_{SBS}(\Delta f)P_{pump}L}{1 + (\int g_{SBS}(\Delta f)P_{lase}(\Delta f)d\Delta f/P_{sat})}$$
(S3)

where  $g_{SBS}(\Delta f)$  is the intrinsic SBS gain spectrum,  $P_{pump}$  is the pump power in the absence of pump depletion, *L* is the length of fiber over which the mode experiences gain (i.e. 4 m for a 40 ns pump pulse), and  $P_{sat}$  is the integrated lasing power at which the gain is reduced by a factor of 2. We neglected spontaneous emission and noise as the primary goal here is to evaluate the relationship between the power spectrum of the lasing mode and the sensor response time, rather than precisely model the system.

We first consider a monochromatic Brillouin lasing mode with power spectrum  $P_{lase}(\Delta f) = P_0 \delta(\Delta f - \Delta f_{lase})$ . In this situation, if the gain spectrum shifts by a small amount (e.g. due to strain), the mode will continue to lase at the same frequency as long as it remains above threshold, since the mode does not contain spectral components at other frequencies which could be amplified. In other words, a monochromatic lasing mode would not respond to small changes in the gain spectrum. In a real system, the lasing mode will always have a finite linewidth and gain pulling will eventually shift the mode back to the center of the gain

spectrum. However, this process can be quite inefficient and could severely limit the response time of the Brillouin laser sensor.

Next consider a more realistic system where the lasing mode is described by a Gaussian lineshape with mean frequency,  $f_{lase,0}$ , peak power,  $P_0$ , and linewidth,  $\Gamma$  ( $P_{lase}(\Delta f) = P_0 e^{(\Delta f - f_{lase,0})^2/\Gamma^2}$ ). If the Brillouin frequency suddenly changes, the number of round-trips, M, required for the peak of the lasing spectrum to match the new peak of the gain spectrum can be approximated as

$$M \simeq \frac{\ln(P_{lase}(0)/P_{lase}(f_{lase,0}))}{G_{SBS}(0) - G_{SBS}(f_{lase,0})}$$
(S4)

where  $P_{lase}(0)/P_{lase}(f_{lase,0})$  is the initial ratio of the lasing power at the new peak of the Brillouin gain spectrum to the maximum lasing power and the denominator describes the relative gain experienced at these two frequencies. As an example, a lasing mode with  $\Gamma = 1.3 \ MHz$  will require M=1000 cycles to shift to the center of the gain spectrum after a sudden shift of 2 MHz at a typical gain of  $e^{G_{SBS}(0)} = 2$ . Even if we assume a high gain of  $e^{G_{SBS}(0)} = 100$ , M=100 cycles would be required to track a shift of 2 MHz. This simple model indicates that the sensor bandwidth can be severely limited in the case of narrowband lasing.

In this work, the spectral content of the lasing mode is also affected by the temporal modulation introduced by EOM<sub>2</sub> after each round trip. Since this time-domain modulation is equivalent to a convolution in the frequency domain, this process ensures that the laser spectrum is spread over a wide bandwidth. Thus a single round trip in the lasing cavity involves gain, loss, and convolution with the Fourier transform of the intensity modulation,  $I(\Delta f)$ . We can then describe the lasing spectrum after the  $m^{th}$  round trip as:

$$P_{lase}(\Delta f, m) = \int \left[ \alpha e^{G_{SBS}(\delta f)} P_{lase}(\delta f, m-1) \right] I(\Delta f - \delta f) d\delta f$$
(S5)

The intensity modulation term essentially describes the redistribution of lasing power from the frequency  $\delta f$  to  $\Delta f$ . We used this expression to simulate the evolution of the lasing spectrum after a sudden shift in the Brillouin gain spectrum. We repeated the simulation for lasing modes exposed to a Gaussian shaped temporal modulation with FWHM varying from 40 ns to 320 ns. The evolution in the peak of the lasing spectrum is shown in Fig. S2(a) after the Brillouin spectrum was shifted by 2 MHz. In each case, the lasing frequency eventually converges to the center of the gain spectrum ( $\Delta f = 0$ ); however, the transition time is considerably faster for lasing modes that are modulated by shorter pulses. We found that a mode modulated by 40 ns pulses returned to within 10 kHz of the center of the gain spectrum after 15 round trips, while a mode modulated by 320 ns pulses required 88 round trips. Thus, the intra-cavity modulation introduced by EOM<sub>2</sub> not only helped to control mode competition, but also increased the sensor bandwidth.



**Fig. S2** (a) Evolution of the center frequency of a lasing mode after the Brillouin frequency shifts by 2 MHz (i.e. from 2 MHz to 0 MHz). The lasing modes that were modulated with shorter pulses converged more rapidly to the center of the shifted gain spectrum. (b) The evolution of the power spectrum using 40 ns modulation. The broad transform limit allows the mode to quickly return to the center of the gain spectrum. **c** The evolution of a mode using 320 ns

modulation. The relatively narrow spectrum requires considerably longer to adjust to a change in the Brillouin resonance.

## Mapping time to position

Mapping measurement time to sensor position was performed using the same conversion as in standard BOTDA or BOTDR. The position of the interaction,  $\Delta z$ , is given by  $\Delta z = (\Delta t - t_0)c/(2n_{eff})$  where  $\Delta t$  is time,  $n_{eff}$  is the effective index of the fiber, and  $t_0$  is the time it takes for the light to propagate from  $\Delta z = 0$  to the detector.

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